

## 5.3

## Connecting Pieces Piecewise Functions

### LEARNING GOALS

In this lesson, you will:

- Write a piecewise function to model data.
- Graph a piecewise function.
- Determine intervals for a piecewise function to best model data.

### KEY TERM

- piecewise function

Some of the most popular children's books from the 1980s and 1990s had an interesting format: the reader controlled the action of the story! At various key moments throughout the text, the reader was given an opportunity to make a decision about the main character's next move. Each choice led to a different outcome.

For example, in a dragon adventure, the reader may have to decide whether the knight should run and hide from a dragon, or grab a sword and try to slay the beast. One set of conditions led to one outcome, while another set of conditions led to a different outcome.

This idea is fairly common today, but at the time it was revolutionary for the same book to have multiple story lines and endings.

Have you ever read a book like this? If so, what did you like or dislike about it?

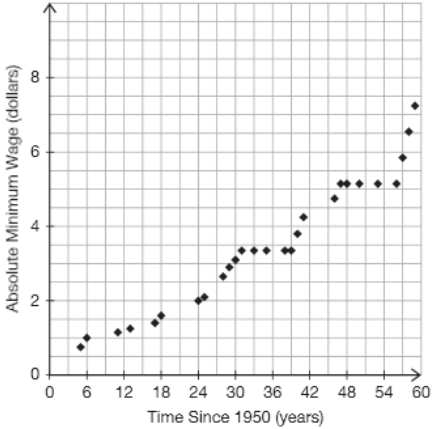
**PROBLEM 1** Keep It to a Minimum, Revisited



Recall the minimum wage problem from the previous lesson.

The table shows the absolute minimum wage during various years. A scatter plot of this data is also shown.

Time Since 1950 (years)	Absolute Minimum Wage (dollars)
5	0.75
6	1.00
11	1.15
13	1.25
17	1.40
18	1.60
24	2.00
25	2.10
28	2.65
29	2.90
30	3.10
31	3.35
33	3.35
35	3.35
38	3.35
39	3.35
40	3.80
41	4.25
46	4.75
47	5.15
48	5.15
50	5.15
53	5.15
56	5.15
57	5.85
58	6.55
59	7.25



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Sometimes, a single polynomial function is not the best model for a set of data. Analyze the graph of the minimum wage data. Instead of using a single polynomial function to model this data, consider separating the data into “pieces,” where each piece is modeled by a single polynomial function.

To determine the pieces, look for breaks in the patterns that you see in the data. For example, the data in one part of the graph may appear to be linear, but then it may appear to be cubic in another part of the graph. Therefore, you can model these two parts of the graph with two different polynomial functions.

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The number of pieces, or functions, that model the data can vary. For example, one person may look at the data and determine that it can be represented with two polynomial functions, while another person may see three, four, or even more functions.

Let's model the absolute minimum wage data by dividing it into five pieces, where each piece is modeled by a different polynomial function.

1. How do you think the data should be divided so that there are five pieces? Circle each piece on the given graph.



2. Consider the data from the years 1955 through 1981.

- a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.

Your graphing calculator is limited in that it can only calculate linear, quadratic, cubic, and quartic polynomial regressions. So, choose one of these for each regression equation.

- b. Write the regression equation that best models the data over this interval.
- c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.

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3. Consider the data after the year 1981 and before the year 1989.

- a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.
- b. Determine a regression equation for this data over this interval.
- c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.

4. Consider the data from the years 1989 through 1997.
  - a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.
  - b. Determine a regression equation for this data over this interval.
  - c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.
5. Consider the data after the year 1997 and before the year 2006.
  - a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.
  - b. Determine a regression equation for this data over this interval.
  - c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.
6. Consider the data from the years 2006 through 2009.
  - a. Describe the type of polynomial function that best models the data over this interval. Explain your reasoning.

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b. Determine a regression equation for this data over this interval.

c. What is the coefficient of determination for the regression equation? Is the model you chose a good fit for the data over this interval? Explain why or why not.

The year 1955 is represented by  $x = 5$ , not  $x = 1955$ . Remember this when you write each domain.



7. Write the equation of the function  $f(x)$ , where  $f(x)$  includes each regression equation you used to model the absolute minimum wage data from the years 1955 through 2009. Write each equation on the line before the comma and write its corresponding domain on the line after the comma. Then, use a graphing calculator to sketch the graph of this function on the scatter plot at the beginning of the problem.

$f(x) = \left\{ \begin{array}{ll} \rule{1.5cm}{0.4pt}, & \rule{1.5cm}{0.4pt} \\ \rule{1.5cm}{0.4pt}, & \rule{1.5cm}{0.4pt} \\ \rule{1.5cm}{0.4pt}, & \rule{1.5cm}{0.4pt} \\ \rule{1.5cm}{0.4pt}, & \rule{1.5cm}{0.4pt} \\ \rule{1.5cm}{0.4pt}, & \rule{1.5cm}{0.4pt} \end{array} \right.$

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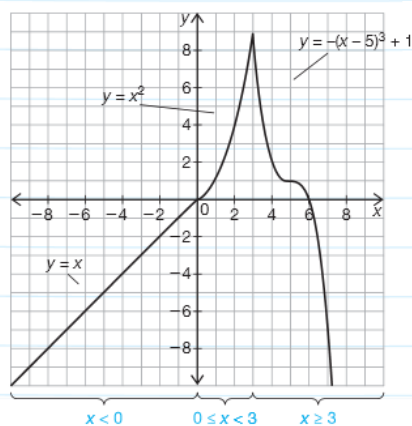
8. Explain why the function you wrote in Question 7 is a better fit for the data than a single linear, quadratic, cubic, or quartic function.



You have just written the equation for a *piecewise function*. A **piecewise function** includes different functions that represent different parts of the domain.

A piecewise function and its graph are shown.

$$f(x) = \begin{cases} x, & x < 0 \\ x^2, & 0 \leq x < 3 \\ -(x-5)^3 + 1, & x \geq 3 \end{cases}$$



Notice the domain is the set of real numbers broken into three different parts.





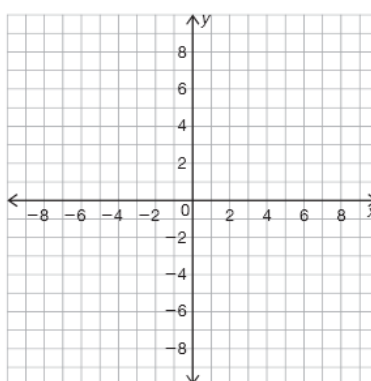
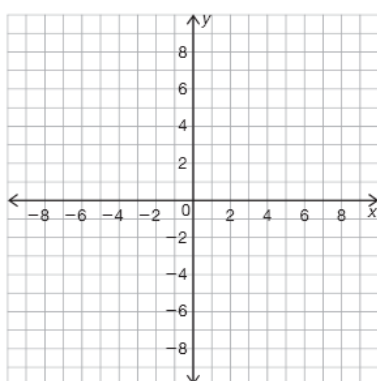
9. Sketch each piecewise function.

Pay attention to whether the endpoints are included or not included for each part of the piecewise function.



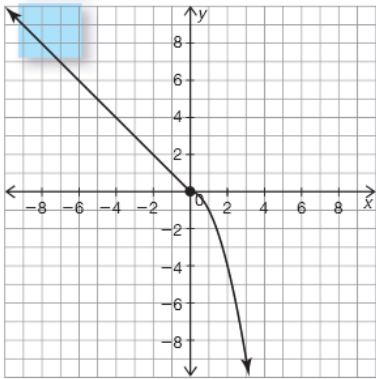
$$\text{a. } g(x) = \begin{cases} \frac{1}{2}x + 1, & x < 4 \\ -(x - 4)^2 + 3, & x \geq 4 \end{cases}$$

$$\text{b. } t(x) = \begin{cases} x, & x < -2 \\ x^4 - 25x^2, & -2 \leq x \leq 2 \\ 2, & x > 2 \end{cases}$$



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10. Billie, Kyle, and Avery were each asked to write the piecewise function to represent the graph shown.



Kyle

$$f(x) = \begin{cases} -x, & x < 0 \\ -x^2, & x \geq 0 \end{cases}$$

Avery

$$f(x) = \begin{cases} -x, & x \leq 0 \\ -x^2, & x > 0 \end{cases}$$

Billie

$$f(x) = \begin{cases} -x, & x \leq 0 \\ -x^2, & x \geq 0 \end{cases}$$

Does analyzing a graph without a scenario change the way you write the function?



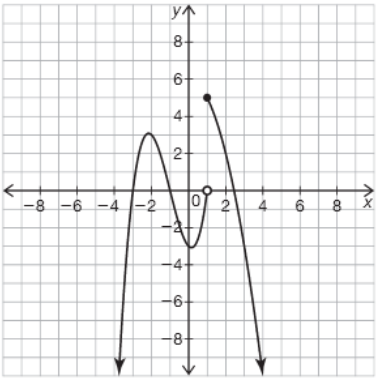
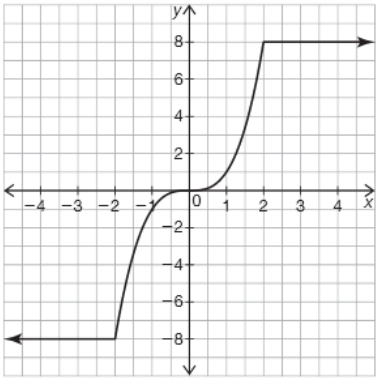
Who's correct? Explain your reasoning.



11. Write the equation for each piecewise function given its graph.

a.  $h(x) =$

b.  $b(x) =$



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PROBLEM 2 Salinity Now! Salinity Now!



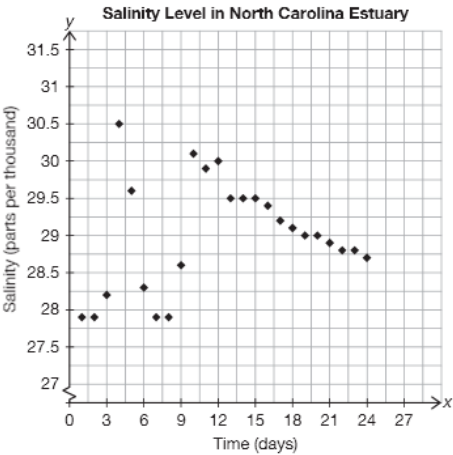
Salinity is the measure of saltiness, or dissolved salt content in water. Salinity in an estuary changes due to location, tidal functions, seasonal weather changes, and volume of freshwater runoff. Ecologists routinely measure salinity in estuaries because of its impact on plants, animals, and people. Too much salinity can reduce vegetation in surrounding areas.

The table shows the salinity levels in an estuary in North Carolina over a period of 24 days. A scatter plot of this data is also shown.

Time (days)	1	2	3	4	5	6	7	8
Salinity (parts per thousand)	27.9	27.9	28.2	30.5	29.6	28.3	27.9	27.9

Time (days)	9	10	11	12	13	14	15	16
Salinity (parts per thousand)	28.6	30.1	29.9	30	29.5	29.5	29.5	29.4

Time (days)	17	18	19	20	21	22	23	24
Salinity (parts per thousand)	29.2	29.1	29.0	29.0	28.9	28.8	28.8	28.7



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1. Consider the data for the first ten days. Determine the regression equation that is the best fit for the data over this interval. Explain your reasoning.

2. Consider the data after the tenth day. Determine the regression equation that is the best fit for the data over this interval. Explain your reasoning.

3. Use your answers to Questions 1 and 2 to write a piecewise function that models the salinity over the 24-day period. Then, graph the function on the scatter plot.

4. Predict the salinity of the estuary on the 30<sup>th</sup> day. Does your prediction seem reasonable?



5. Predict the salinity of the estuary 5 days before the data in the table was recorded. Does your prediction seem reasonable?

**PROBLEM 3** Fill It Up!



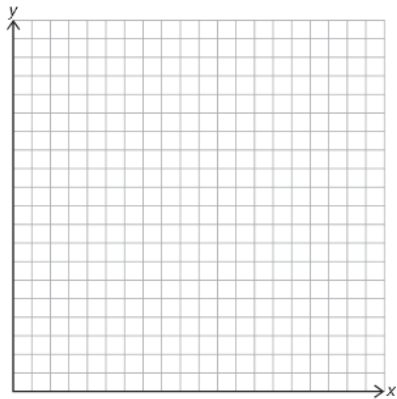
The table shows the average price of a gallon of regular unleaded gas from the years 1980 through 2008.

Years Since 1980	0	1	2	3	4	5	6	7	8	9
Average Gas Price (dollars)	1.25	1.38	1.30	1.24	1.21	1.20	0.93	0.95	0.95	1.02

Years Since 1980	10	11	12	13	14	15	16	17	18	19
Average Gas Price (dollars)	1.16	1.14	1.13	1.11	1.11	1.15	1.23	1.23	1.06	1.17


Years Since 1980	20	21	22	23	24	25	26	27	28
Average Gas Price (dollars)	1.51	1.46	1.36	1.59	1.88	2.30	2.59	2.80	3.27

1. Create a scatter plot of the data on the grid shown.



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2. Describe the type of function(s) that best models this data. Explain your reasoning.

3. Consider using a piecewise function to model this data. Determine the intervals for the domain, and the type of polynomial function for each interval. Explain your reasoning.
4. Write a piecewise function to model the data. Then, graph the piecewise function on the grid in Question 1.
-  5. Use your piecewise function to predict the price of gas in the year 2020. Does your prediction seem reasonable? Explain your reasoning.

**PROBLEM 4** Live Long and Strong



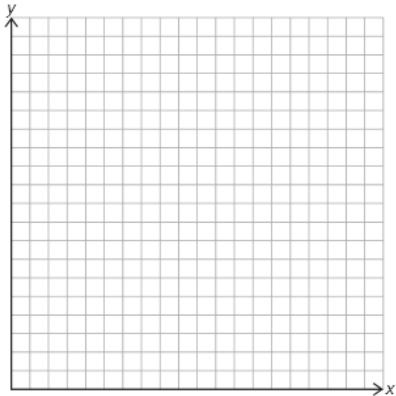
Life expectancy is a prediction of the number of years that a person will live. Life expectancies often vary significantly over time and across different groups such as country, gender, and race.

The table shows the average life expectancy of a person from the years 1910 through 1920.

Years Since 1910	0	1	2	3	4	5
Life Expectancy (years)	50.6	52.7	53.7	52.7	54.4	54.7

Years Since 1910	6	7	8	9	10
Life Expectancy (years)	52.0	51.2	39.4	54.8	54.1

- 1. Create a scatter plot of this data on the grid shown.



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2. Write a piecewise function to model the data. Then, graph the piecewise function on the grid. Explain your reasoning.



3. Write a brief report that explains the patterns shown by the data in terms of life expectancy from 1910 through 1920. Do some research and use facts to support your claims.

**5****Talk the Talk**

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Write a brief summary about what you've learned about using piecewise functions to model real-world data. Include advantages and disadvantages of using a piecewise function instead of a single function type to model data.



Be prepared to share your solutions and methods.